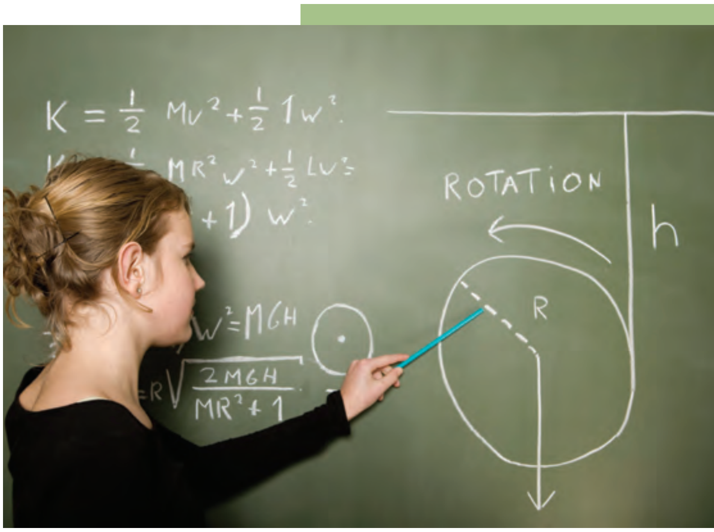
Practice 5 Using Mathematics and Computational Thinking

Mathematics and computational tools are central to science and engineering. Mathematics enables the numerical representation of variables, the symbolic representation of relationships between physical entities, and the prediction of outcomes. Mathematics provides powerful models for describing and predicting such phenomena as atomic structure, gravitational forces, and quantum mechanics.



Since the mid-20th century, computational theories, information and computer technologies, and algorithms have revolutionized virtually all scientific and engineering fields. These tools and strategies allow scientists and engineers to collect and analyze large data sets, search for distinctive patterns, and identify relationships and significant features in ways that were previously impossible. They also provide powerful new techniques for employing mathematics to model complex phenomena—for example, the circulation of carbon dioxide in the atmosphere and ocean.

Mathematics and computation can be powerful tools when brought to bear in a scientific investigation. Mathematics serves pragmatic functions as a tool—both a communicative function, as one of the languages of science, and a structural function, which allows for logical deduction. Mathematics enables ideas to be expressed in a precise form and enables the identification of new ideas about the physical world. For example, the concept of the equivalence of mass and energy emerged from the mathematical analysis conducted by Einstein, based on the premises of special relativity. The contemporary understanding of electromagnetic waves emerged from Maxwell's mathematical analysis of the behavior of electric and magnetic fields. Modern theoretical physics is so heavily imbued with mathematics that it would make no sense to try to divide it into mathematical and nonmathematical parts. In much of modern science, predictions and inferences have a probabilistic nature, so understanding the mathematics of probability and of statistically derived inferences is an important part of understanding science.

Computational tools enhance the power of mathematics by enabling calculations that cannot be carried out analytically. For example, they allow the development of simulations, which combine mathematical representations of

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multiple underlying phenomena to model the dynamics of a complex system. Computational methods are also potent tools for visually representing data, and they can show the results of calculations or simulations in ways that allow the exploration of patterns.

Engineering, too, involves mathematical and computational skills. For example, structural engineers create mathematical models of bridge and building designs, based on physical laws, to test their performance, probe their structural limits, and assess whether they can be completed within acceptable budgets. Virtually any engineering design raises issues that require computation for their resolution.

Although there are differences in how mathematics and computational thinking are applied in science and in engineering, mathematics often brings these two fields together by enabling engineers to apply the mathematical form of scientific theories and by enabling scientists to use powerful information technologies designed by engineers. Both kinds of professionals can thereby accomplish investigations and analyses and build complex models, which might otherwise be out of the question.

Mathematics (including statistics) and computational tools are essential for data analysis, especially for large data sets. The abilities to view data from different perspectives and with different graphical representations, to test relationships between variables, and to explore the interplay of diverse external conditions all require mathematical skills that are enhanced and extended with computational skills.

GOALS

By grade 12, students should be able to

- Recognize dimensional quantities and use appropriate units in scientific applications of mathematical formulas and graphs.
- Express relationships and quantities in appropriate mathematical or algorithmic forms for scientific modeling and investigations.

- Recognize that computer simulations are built on mathematical models that incorporate underlying assumptions about the phenomena or systems being studied.
- Use simple test cases of mathematical expressions, computer programs, or simulations—that is, compare their outcomes with what is known about the real world—to see if they "make sense."
- Use grade-level-appropriate understanding of mathematics and statistics in analyzing data.

PROGRESSION

Increasing students' familiarity with the role of mathematics in science is central to developing a deeper understanding of how science works. As soon as students learn to count, they can begin using numbers to find or describe patterns in nature. At appropriate grade levels, they should learn to use such instruments as rulers, protractors, and thermometers for the measurement of variables that are best represented by a continuous numerical scale, to apply mathematics to interpolate values, and to identify features—such as maximum, minimum, range, average, and median—of simple data sets.

A significant advance comes when relationships are expressed using equalities first in words and then in algebraic symbols—for example, shifting from distance traveled equals velocity multiplied by time elapsed to $s = vt$. Students should have opportunities to explore how such symbolic representations can be used to represent data, to predict outcomes, and eventually to derive further relationships using mathematics. Students should gain experience in using computers to record measurements taken with computer-connected probes or instruments, thereby recognizing how this process allows multiple measurements to be made rapidly and recurrently. Likewise, students should gain experience in using computer programs to transform their data between various tabular and graphical forms, thereby aiding in the identification of patterns.

Students should thus be encouraged to explore the use of computers for data analysis, using simple data sets, at an early age. For example, they could use spreadsheets to record data and then perform simple and recurring calculations from those data, such as the calculation of average speed from measurements of positions at multiple times. Later work should introduce them to the use of mathematical relationships to build simple computer models, using appropriate supporting programs or information and computer technology tools. As students progress in their understanding of mathematics and computation, at every level the science classroom should be a place where these tools are progressively exploited.